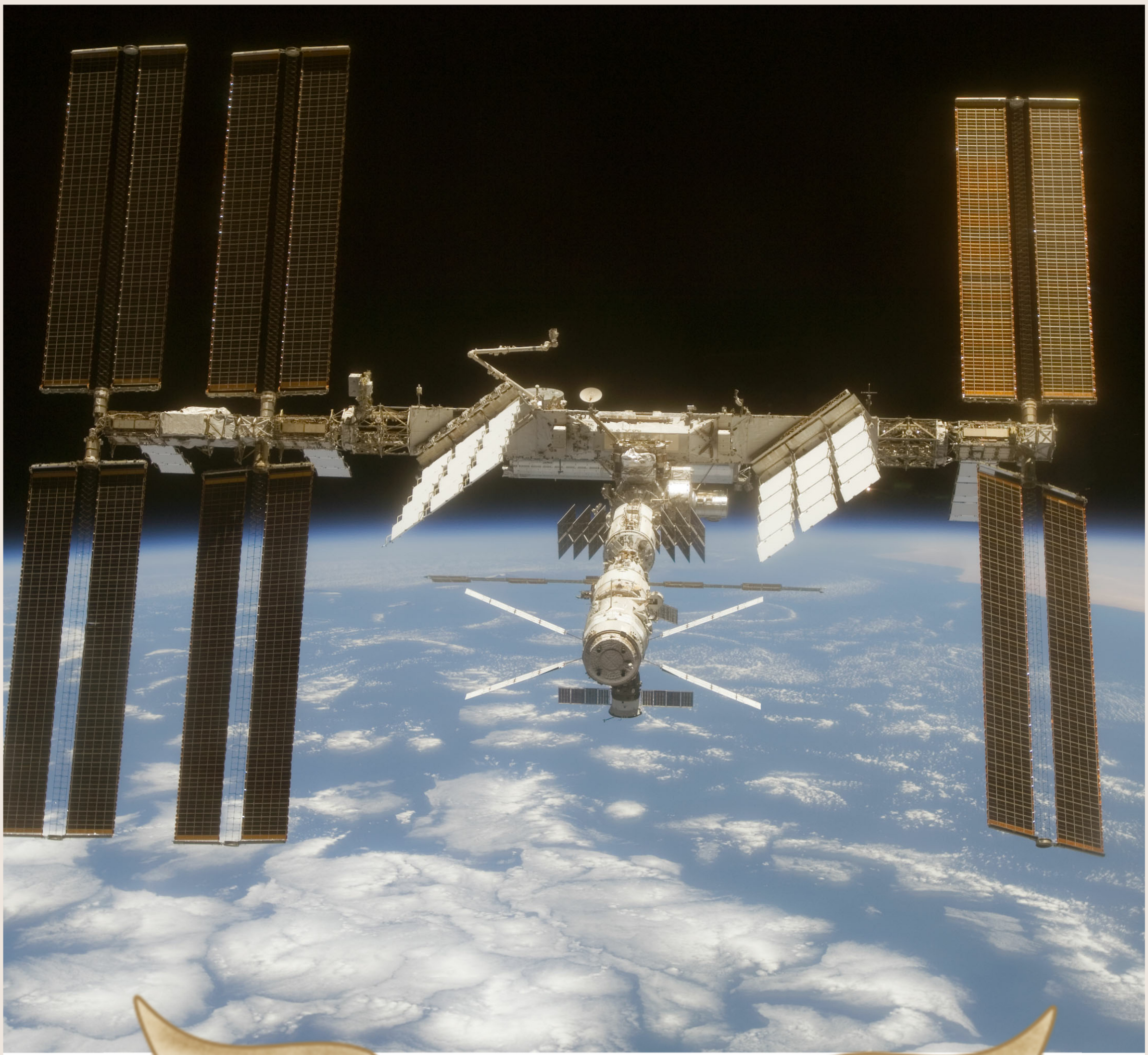
Year 12 Extension 1 Maths

Calculus n noun
1 (plural calculuses) (also infinitesimal calculus) the branch of mathematics concerned with the determination and properties of derivatives and integrals of functions, by methods based on the summation of infinitesimal differences.
a particular method or system of calculation or reasoning.

ORIGIN
C17: from Latin, literally 'small pebble'.

"The mind that opens to a new idea never returns to its original size."
Albert Einstein



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Topic: Proof

Outcomes

A student:

- applies techniques involving proof or calculus to model and solve problems ME12-1
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Topic Focus

The topic Proof involves the communication and justification of an argument for a mathematical statement in a clear, concise and precise manner.

A knowledge of proof enables a level of reasoning, justification and communication that is accurate, concise and precise.

The study of proof is important in developing students' ability to reason, justify, communicate and critique mathematical arguments and statements necessary for problem-solving and generalising patterns.

Subtopics

ME-P1 Proof by Mathematical Induction

Proof

ME-P1 Proof by Mathematical Induction

Outcomes

A student:

- applies techniques involving proof or calculus to model and solve problems ME12-1
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Subtopic Focus

The principal focus of this subtopic is to explore and to develop the use of the technique of proof by mathematical induction to prove results. Students are introduced to mathematical induction for a limited range of applications so that they have time to develop confidence in its use.

Students develop the use of formal mathematical language and argument to prove the validity of given situations using inductive reasoning. The logical sequence of steps in the proof technique needs to be understood and carefully justified, thus encouraging clear and concise communication which is useful both in further study of mathematics and in life.

Content

Students:

- understand the nature of inductive proof, including the 'initial statement' and the inductive step (ACMSM064)

Students:

- prove results using mathematical induction
  - prove results for sums, for example $1 + 4 + 9 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n (ACMSM065)
  - prove divisibility results, for example $3^{2n} - 1$ is divisible by 8 for any positive integer n (ACMSM066)

Students:

- identify errors in false 'proofs by induction', such as cases where only one of the required two steps of a proof by induction is true, and understand that this means that the statement has not been proved

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Students:

- recognise situations where proof by mathematical induction is not appropriate

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Topic: Vectors

Outcomes

A student:

- applies concepts and techniques involving vectors and projectiles to solve problems ME12-2
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Topic Focus

The topic Vectors involves mathematical representation of a quantity with magnitude and direction and its geometrical depiction. This topic provides a modern language and approach to explore and explain a range of object behaviours in a variety of contexts from theoretical or real-life scenarios.

A knowledge of vectors enables the understanding of the behaviour of objects in two dimensions and ways in which this behaviour can be expressed, including the consideration of position, displacement and movement.

The study of vectors is important in developing students' understanding of an object's representation and behaviour in two dimensions using a variety of notations, and how to use these notations effectively to explore the geometry of a situation. Vectors are used in many fields of study, including engineering, structural analysis and navigation.

Subtopics

ME-V1 Introduction to Vectors

Vectors

ME-V1 Introduction to Vectors

Outcomes

A student:

- applies concepts and techniques involving vectors and projectiles to solve problems ME12-2
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Subtopic Focus

The principal focus of this subtopic is to introduce the concept of vectors in two dimensions, use them to represent quantities with magnitude and direction, and understand that this representation can allow for the exploration of situations such as geometrical proofs.

Students develop an understanding of vector notations and how to manipulate vectors to allow geometrical situations to be explored further. The example of projectile motion as an application of vectors is then introduced. These concepts are explored further in the Mathematics Extension 2 course.

Content

V1.1: Introduction to vectors

Students:

- define a vector as a quantity having both magnitude and direction, and examine examples of vectors, including displacement and velocity (ACMSM010)
  - explain the distinction between a position vector and a displacement (relative) vector

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Students:

- define and use a variety of notations and representations for vectors in two dimensions (ACMSM014)
  - use standard notations for vectors, for example: $\underset{\sim}{a}$, $\overrightarrow{AB}$ and $\mathbf{a}$
  - represent vectors graphically in two dimensions as directed line segments
  - define unit vectors as vectors of magnitude 1, and the standard two-dimensional perpendicular unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$
  - express and use vectors in two dimensions in a variety of forms, including component form, ordered pairs and column vector notation

Students:

- perform addition and subtraction of vectors and multiplication of a vector by a scalar algebraically and geometrically, and interpret these operations in geometric terms **AAM**
  - graphically represent a scalar multiple of a vector (ACMSM012)
  - use the triangle law and the parallelogram law to find the sum and difference of two vectors
  - define and use addition and subtraction of vectors in component form (ACMSM017)
  - define and use multiplication by a scalar of a vector in component form (ACMSM018)

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Students:

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V1.2: Further operations with vectors

Students:

- define, calculate and use the magnitude of a vector in two dimensions and use the notation $\left|\underset{\sim}{u}\right|$ for the magnitude of a vector $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j}$
  - prove that the magnitude of a vector, $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j}$, can be found using: $\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$
  - identify the magnitude of a displacement vector $\overrightarrow{AB}$ as being the distance between the points A and B
  - convert a non-zero vector $\underset{\sim}{u}$ into a unit vector $\hat{\underset{\sim}{u}}$ by dividing by its length: $\hat{\underset{\sim}{u}} = \dfrac{\underset{\sim}{u}}{\left|\underset{\sim}{u}\right|}$

Students:

Students:

- define and use the direction of a vector in two dimensions

Students:

Students:

- define, calculate and use the scalar (dot) product of two vectors $\underset{\sim}{u} = x_1 \underset{\sim}{i} + y_1 \underset{\sim}{j}$ and $\underset{\sim}{v} = x_2 \underset{\sim}{i} + y_2 \underset{\sim}{j}$

  AAM

  – apply the scalar product, $\underset{\sim}{u} \cdot \underset{\sim}{v}$, to vectors expressed in component form, where $\underset{\sim}{u} \cdot \underset{\sim}{v} = x_1 x_2 + y_1 y_2$

  – use the expression for the scalar (dot) product, $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$ where θ is the angle between vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ to solve problems

  – demonstrate the equivalence, $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1 x_2 + y_1 y_2$ and use this relationship to solve problems

  – establish and use the formula $\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$

  – calculate the angle between two vectors using the scalar (dot) product of two vectors in two dimensions

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Students:

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Students:

- examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)

Students:

- define and use the projection of one vector onto another (ACMSM022)

Students:

- solve problems involving displacement, force and velocity involving vector concepts in two dimensions (ACMSM023) **AAM**

Students:

- prove geometric results and construct proofs involving vectors in two dimensions including but not limited to proving that: **AAM**
  - the diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)
  - the midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)
  - the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides (ACMSM041)

V1.3: Projectile motion

Students:

- understand the concept of projectile motion, and model and analyse a projectile's path assuming that:
  - the projectile is a point
  - the force due to air resistance is negligible
  - the only force acting on the projectile is the constant force due to gravity, assuming that the projectile is moving close to the Earth's surface

Students:

- model the motion of a projectile as a particle moving with constant acceleration due to gravity and derive the equations of motion of a projectile **AAM**
  - represent the motion of a projectile using vectors
  - recognise that the horizontal and vertical components of the motion of a projectile can be represented by horizontal and vertical vectors
  - derive the horizontal and vertical equations of motion of a projectile
  - understand and explain the limitations of this projectile model

Students:

- use equations for horizontal and vertical components of velocity and displacement to solve problems on projectiles

Students:

- apply calculus to the equations of motion to solve problems involving projectiles (ACMSM115) **AAM**

Topic: Trigonometric Functions

Outcomes

A student:

- applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations ME12-3
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Topic Focus

The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations. It extends to include the exploration of both algebraic and geometric methods to solve trigonometric problems.

A knowledge of trigonometric functions enables students to manipulate trigonometric expressions to prove identities and solve equations.

The study of trigonometric functions is important in developing students’ understanding of the connections between algebraic and graphical representations and how this can be applied to solve problems from theoretical or real-life scenarios, for example involving waves and signals.

Subtopics

ME-T3 Trigonometric Equations

Trigonometric Functions

ME-T3 Trigonometric Equations

Outcomes

A student:

- › applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations ME12-3
- › chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- › evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Subtopic Focus

The principal focus of this subtopic is to consolidate and extend students' knowledge in relation to solving trigonometric equations and to apply this knowledge to practical situations.

Students develop complex algebraic manipulative skills and fluency in applying trigonometric knowledge to a variety of situations. Trigonometric expressions and equations provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources, and alternating current.

Content

Students:

- convert expressions of the form $a \cos x + b \sin x$ to $R \cos(x \pm \alpha)$ or $R \sin(x \pm \alpha)$ and apply these to solve equations of the form $a \cos x + b \sin x = c$, sketch graphs and solve related problems (ACMSM048)

Students:

- solve trigonometric equations requiring factorising and/or the application of compound angle, double angle formulae or the t-formulae

Students:

- prove and apply other trigonometric identities, for example $\cos 3x = 4\cos^3 x - 3\cos x$ (ACMSM049)

Students:

- solve trigonometric equations and interpret solutions in context using technology or otherwise

Topic: Calculus

Outcomes

A student:

- › applies techniques involving proof or calculus to model and solve problems ME12-1
- › uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution ME12-4
- › chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- › evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Topic Focus

The topic Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It involves the development of analytic and numeric integration techniques and the use of these techniques in solving problems.

The study of calculus is important in developing students' knowledge, understanding and capacity to operate with and model situations involving change, and to use algebraic and graphical techniques to describe and solve problems and to predict future outcomes with relevance to, for example science, engineering, finance, economics and the construction industry.

Subtopics

ME-C2 Further Calculus Skills
ME-C3 Applications of Calculus

Calculus

ME-C2 Further Calculus Skills

Outcomes

A student:

- applies techniques involving proof or calculus to model and solve problems ME12-1
- uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution ME12-4
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Subtopic Focus

The principal focus of this subtopic is to further develop students' knowledge, skills and understanding relating to differentiation and integration techniques.

Students develop an awareness and understanding of the interconnectedness of topics across the syllabus, and the fluency that can be obtained in the use of calculus techniques. Later studies in mathematics place prime importance on familiarity and confidence in a variety of calculus techniques as these are used in many different fields.

Content

Students:

- find and evaluate indefinite and definite integrals using the method of integration by substitution, using a given substitution
  - change an integrand into an appropriate form using algebra

Students:

- prove and use the identities $\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$ and $\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$ to solve problems

Students:

- solve problems involving $\int \sin^2 nx \, dx$ and $\int \cos^2 nx \, dx$

Students:

- find derivatives of inverse functions by using the relationship $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

Students:

- solve problems involving the derivatives of inverse trigonometric functions

Students:

- integrate expressions of the form $\frac{1}{\sqrt{a^2-x^2}}$ or $\frac{a}{a^2+x^2}$ (ACMSM121)

Calculus

ME-C3 Applications of Calculus

Outcomes

A student:

- applies techniques involving proof or calculus to model and solve problems ME12-1
- uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution ME12-4
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Subtopic Focus

The principal focus of this subtopic is to develop an understanding of applications of calculus in a practical context, including the more accessible kinds of differential equations and volumes of solids of revolution, to solve problems.

Students develop an awareness and understanding of the use of differential equations which arise when the rate of change in one quantity with respect to another can be expressed in mathematical form. The study of differential equations has important applications in science, engineering, finance, economics and broader applications in mathematics.

Content

C3.1: Further area and volumes of solids of revolution

Students:

- calculate area of regions between curves determined by functions (ACMSM124)

Students:

- sketch, with and without the use of technology, the graph of a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis **AAM**

Students:

- calculate the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis, with and without the use of technology (ACMSM125) **AAM**

Students:

- determine the volumes of solids of revolution that are formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts **AAM**

C3.2: Differential equations

Students:

- recognise that an equation involving a derivative is called a differential equation

Students:

- recognise that solutions to differential equations are functions and that these solutions may not be unique

Students:

- sketch the graph of a particular solution given a direction field and initial conditions
  - form a direction field (slope field) from simple first-order differential equations
  - recognise the shape of a direction field from several alternatives given the form of a differential equation, and vice versa
  - sketch several possible solution curves on a given direction field

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Students:

- solve simple first-order differential equations (ACMSM130)
  - solve differential equations of the form $\frac{dy}{dx} = f(x)$
  - solve differential equations of the form $\frac{dy}{dx} = g(y)$
  - solve differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables

Students:

- recognise the features of a first-order linear differential equation and that exponential growth and decay models are first-order linear differential equations, with known solutions

Students:

- model and solve differential equations including but not limited to the logistic equation that will arise in situations where rates are involved, for example in chemistry, biology and economics (ACMSM132) **AAM**

Topic: Statistical Analysis

Outcomes

A student:

- applies appropriate statistical processes to present, analyse and interpret data ME12-5
- chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
- evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Topic Focus

The topic Statistical Analysis involves the exploration, display and interpretation of data via modelling to identify and communicate key information.

A knowledge of statistical analysis enables careful interpretation of situations and an awareness of the contributing factors when presented with information by third parties, including its possible misrepresentation.

The study of statistical analysis is important in developing students' ability to consider the level of reliability that can be applied to the analysis of current situations and to predict future outcomes. It supports the development of understanding of how conclusions drawn from data can be used to inform decisions made by groups such as scientific investigators, business people and policy-makers.

Subtopics

ME-S1 The Binomial Distribution

Statistical Analysis

ME-S1 The Binomial Distribution

Outcomes

A student:

› applies appropriate statistical processes to present, analyse and interpret data ME12-5
› chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
› evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

Subtopic Focus

The principal focus of this subtopic is to develop an understanding of binomial random variables and their uses in modelling random processes involving chance and variation.

Students develop an understanding of binomial distributions and associated statistical analysis methods and their use in modelling binomial events. Binomial probabilities and the binomial distribution are used to model situations where only two outcomes are possible. The use of the binomial distribution and binomial probability has many applications, including medicine and genetics.

Content

S1.1: Bernoulli and binomial distributions

Students:

- use a Bernoulli random variable as a model for two-outcome situations (ACMMM143)
  - identify contexts suitable for modelling by Bernoulli random variables (ACMMM144)

Students:

- use Bernoulli random variables and their associated probabilities to solve practical problems (ACMMM146) **AAM**
  - understand and apply the formulae for the mean, $E(X) = \bar{x} = p$, and variance, $\mathrm{Var}(X) = p(1-p)$, of the Bernoulli distribution with parameter p, and X defined as the number of successes (ACMMM145)

Students:

- understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial (ACMMM147)
  - calculate the expected frequencies of the various possible outcomes from a series of Bernoulli trials

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Students:

- use binomial distributions and their associated probabilities to solve practical problems (ACMMM150) **AAM**
  - identify contexts suitable for modelling by binomial random variables (ACMMM148)
  - identify the binomial parameter p as the probability of success
  - apply the formulae for probabilities $P(X = r) = {}^nC_r p^r (1-p)^{n-r}$ associated with the binomial distribution with parameters n and p and understand the meaning of ${}^nC_r$ as the number of ways in which an outcome with r successes can occur
  - understand and apply the formulae for the mean, $E(X) = \bar{x} = np$, and the variance, $\mathrm{Var}(X) = np(1-p)$, of a binomial distribution with parameters n and p

S1.2: Normal approximation for the sample proportion

Students:

- use appropriate graphs to explore the behaviour of the sample proportion on collected or supplied data **AAM**
  - understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples (ACMMM174)

Students:

- explore the behaviour of the sample proportion using simulated data **AAM**
  - examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)

Students:

- understand and use the normal approximation to the distribution of the sample proportion and its limitations **AAM**

Students: