Advanced Mathematics

Year 12

Customer:

"How much is a large order of Fibonaccos?"

Cashier:

"It's the price of a small order plus the price of a medium order."

The mind is not a
vessel to be filled,
but a fire to be kindled.
Plutarch

Topic: Functions

Outcomes

A student:

- uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Topic Focus

The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities.

A knowledge of functions enables students to discover, recognise and generalise connections between algebraic and graphical representations of the same expression and to describe interactions between dependent and independent variables.

The study of functions is important in developing students' ability to find and recognise connections and patterns, to communicate concisely and precisely, to use algebraic techniques and manipulations to describe and solve problems, and predict future outcomes in areas such as finance, economics and weather.

Subtopics

MA-F2 Graphing Techniques

Functions

MA-F2 Graphing Techniques

Outcomes

A student:

- uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to become more familiar with key features of graphs of functions, as well as develop an understanding of and use of the effect of basic transformations of these graphs to explain graphical behaviour.

Students develop an understanding of transformations from a graphical and algebraic approach, including the use of technology, and thus develop a deeper understanding of the properties of functions. As graphing software becomes more widely accessible, skills in reading scales and interpreting magnification effects become essential.

Content

Students:

- apply transformations to sketch functions of the form $y = kf(a(x+b)) + c$, where $f(x)$ is a polynomial, reciprocal, absolute value, exponential or logarithmic function and a, b, c and k are constants
  – examine translations and the graphs of $y = f(x) + c$ and $y = f(x+b)$ using technology
  – examine dilations and the graphs of $y = kf(x)$ and $y = f(ax)$ using technology
  – recognise that the order in which transformations are applied is important in the construction of the resulting function or graph

Students:

Students:

- use graphical methods with supporting algebraic working to solve a variety of practical problems involving any of the functions within the scope of this syllabus, in both real-life and abstract contexts **AAM**
  - select and use an appropriate method to graph a given function, including finding intercepts, considering the sign of $f(x)$ and using symmetry
  - determine asymptotes and discontinuities where appropriate (vertical and horizontal asymptotes only)
  - determine the number of solutions of an equation by considering appropriate graphs
  - solve linear and quadratic inequalities by sketching appropriate graphs

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Students:

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Topic: Trigonometric Functions

Outcomes

A student:

- › uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1
- › applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs MA12-5
- › chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- › constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Topic Focus

The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations.

A knowledge of trigonometric functions enables the solving of practical problems involving the manipulation of trigonometric expressions to model behaviour of naturally occurring periodic phenomena such as waves and signals and to predict future outcomes.

Study of trigonometric functions is important in developing students' understanding of periodic functions. Utilising the properties of periodic functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

Subtopics

MA-T3 Trigonometric Functions and Graphs

Trigonometric Functions

MA-T3 Trigonometric Functions and Graphs

Outcomes

A student:

› uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1
› applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs MA12-5
› chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
› constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to explore the key features of the graphs of trigonometric functions and to understand and use basic transformations to solve trigonometric equations.

Students develop an understanding of the way that graphs of trigonometric functions change when the functions are altered in a systematic way. This is important in understanding how mathematical models of real-world phenomena can be developed.

Content

Students:

- examine and apply transformations to sketch functions of the form $y = kf(a(x + b)) + c$, where a, b, c and k are constants, in a variety of contexts, where $f(x)$ is one of $\sin x$, $\cos x$ or $\tan x$, stating the domain and range when appropriate
  - use technology or otherwise to examine the effect on the graphs of changing the amplitude (where appropriate), $y = kf(x)$, the period, $y = f(ax)$, the phase, $y = f(x + b)$, and the vertical shift, $y = f(x) + c$
  - use k, a, b, c to describe transformational shifts and sketch graphs

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Students:

- solve trigonometric equations involving functions of the form $kf(a(x + b)) + c$, using technology or otherwise, within a specified domain **AAM**

Students:

- use trigonometric functions of the form $kf(a(x+b))+c$ to model and/or solve practical problems involving periodic phenomena **AAM**

Students:

Topic: Calculus

Outcomes

A student:

- applies calculus techniques to model and solve problems MA12-3
- applies appropriate differentiation methods to solve problems MA12-6
- applies the concepts and techniques of indefinite and definite integrals in the solution of problems MA12-7
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Topic Focus

The topic Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It involves the development of two key aspects of calculus, namely differentiation and integration.

The study of calculus is important in developing students’ capacity to operate with and model situations involving change, using algebraic and graphical techniques to describe and solve problems and to predict outcomes in fields such as biomathematics, economics, engineering and the construction industry.

Subtopics

MA-C2 Differential Calculus
MA-C3 Applications of Differentiation
MA-C4 Integral Calculus

Calculus

MA-C2 Differential Calculus

Outcomes

A student:

- applies calculus techniques to model and solve problems MA12-3
- applies appropriate differentiation methods to solve problems MA12-6
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to develop and apply rules for differentiation to a variety of functions.

Students develop an understanding of the interconnectedness of topics from across the syllabus and the use of calculus to help solve problems from each topic. These skills are then applied in the following subtopic on the second derivative in order to investigate applications of the calculus of trigonometric, exponential and logarithmic functions.

Content

C2.1: Differentiation of trigonometric, exponential and logarithmic functions

Students:

- establish the formulae $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions (ACMMM102)

Students:

- calculate derivatives of trigonometric functions

Students:

- establish and use the formula $\frac{d}{dx}(a^x) = (\ln a)a^x$
  - using graphing software or otherwise, sketch and explore the gradient function for a given exponential function, recognise it as another exponential function and hence determine the relationship between exponential functions and their derivatives

Students:

- calculate the derivative of the natural logarithm function $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Students:

- establish and use the formula $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Students:

C2.2: Rules of differentiation

Students:

- apply the product, quotient and chain rules to differentiate functions of the form $f(x)g(x), \frac{f(x)}{g(x)}$ and $f(g(x))$ where $f(x)$ and $g(x)$ are any of the functions covered in the scope of this syllabus, for example $xe^x, \tan x, \frac{1}{x^n}, x \sin x, e^{-x} \sin x$ and $f(ax+b)$ (ACMMM106)
  - use the composite function rule (chain rule) to establish that $\frac{d}{dx}\{e^{f(x)}\} = f'(x)e^{f(x)}$
  - use the composite function rule (chain rule) to establish that $\frac{d}{dx}\{\ln f(x)\} = \frac{f'(x)}{f(x)}$
  - use the logarithmic laws to simplify an expression before differentiating
  - use the composite function rule (chain rule) to establish and use the derivatives of $\sin(f(x))$, $\cos(f(x))$ and $\tan(f(x))$

Students:

Calculus

MA-C3 Applications of Differentiation

Outcomes

A student:

- applies calculus techniques to model and solve problems MA12-3
- applies appropriate differentiation methods to solve problems MA12-6
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to introduce the second derivative, its meanings and applications to the behaviour of graphs and functions, such as stationary points and the concavity of the graph.

Students develop an understanding of the interconnectedness of topics from across the syllabus and the use of calculus to help solve problems such as optimisation, from each topic. The solution of optimisation problems is an important area of applied Mathematics and involves the location of the maximum or minimum values of a function.

Content

C3.1: The first and second derivatives

Students:

- use the first derivative to investigate the shape of the graph of a function
  - deduce from the sign of the first derivative whether a function is increasing, decreasing or stationary at a given point or in a given interval
  - use the first derivative to find intervals over which a function is increasing or decreasing, and where its stationary points are located
  - use the first derivative to investigate a stationary point of a function over a given domain, classifying it as a local maximum, local minimum or neither
  - determine the greatest or least value of a function over a given domain (if the domain is not given, the natural domain of the function is assumed) and distinguish between local and global minima and maxima

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Students:

- define and interpret the concept of the second derivative as the rate of change of the first derivative function in a variety of contexts, for example recognise acceleration as the second derivative of displacement with respect to time (ACMMM108, ACMMM109) **AAM**
  - understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)
  - use the second derivative to determine concavity and the nature of stationary points
  - understand that when the second derivative is equal to 0 this does not necessarily represent a point of inflection

Students:

C3.2: Applications of the derivative

Students:

- use any of the functions covered in the scope of this syllabus and their derivatives to solve practical and abstract problems **AAM**

Students:

- use calculus to determine and verify the nature of stationary points, find local and global maxima and minima and points of inflection (horizontal or otherwise), examine behaviour of a function as $x \to \infty$ and $x \to -\infty$ and hence sketch the graph of the function (ACMMM095)

Students:

- solve optimisation problems for any of the functions covered in the scope of this syllabus, in a wide variety of contexts including but not limited to displacement, velocity, acceleration, area, volume, business, finance and growth and decay **AAM**
  - define variables and construct functions to represent the relationships between variables related to contexts involving optimisation, sketching diagrams or completing diagrams if necessary
  - use calculus to establish the location of local and global maxima and minima, including checking endpoints of an interval if required
  - evaluate solutions and their reasonableness given the constraints of the domain and formulate appropriate conclusions to optimisation problems

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Calculus

MA-C4 Integral Calculus

Outcomes

A student:

- applies calculus techniques to model and solve problems MA12-3
- applies the concepts and techniques of indefinite and definite integrals in the solution of problems MA12-7
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to introduce the anti-derivative or indefinite integral and to develop and apply methods for finding the area under a curve, including the Trapezoidal rule and the definite integral, for a range of functions in a variety of contexts.

Students develop their understanding of how integral calculus relates to area under curves and a further understanding of the interconnectedness of topics from across the syllabus. Geometrical representation assists in understanding the development of this topic, but careful sequencing of the ideas is required so that students can see that integration has many applications, not only in mathematics but also in other fields such as the sciences and engineering.

Content

C4.1: The anti-derivative

Students:

- define anti-differentiation as the reverse of differentiation and use the notation $\int f(x)\,dx$ for anti-derivatives or indefinite integrals (ACMMM114, ACMMM115)

Students:

- recognise that any two anti-derivatives of $f(x)$ differ by a constant

Students:

- establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$, for $n \neq -1$ (ACMMM116)

Students:

- establish and use the formula $\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$ where $n \neq -1$ (the reverse chain rule)

Students:

- establish and use the formulae for the anti-derivatives of $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$

Students:

- establish and use the formulae $\int e^x dx = e^x + c$ and $\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + c$

Students:

- establish and use the formulae $\int \frac{1}{x} dx = \ln|x| + c$ and $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$ for $x \neq 0, f(x) \neq 0$, respectively

Students:

- establish and use the formulae $\int a^x dx = \frac{a^x}{\ln a} + c$

Students:

- recognise and use linearity of anti-differentiation (ACMMM119)
  - examine families of anti-derivatives of a given function graphically

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Students:

- determine indefinite integrals of the form $\int f(ax + b)\, dx$ (ACMMM120)

Students:

- determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$ in a range of practical and abstract applications including coordinate geometry, business and science

C4.2: Areas and the definite integral

Students:

- know that ‘the area under a curve’ refers to the area between a function and the x-axis, bounded by two values of the independent variable and interpret the area under a curve in a variety of contexts **AAM**

Students:

- determine the approximate area under a curve using a variety of shapes including squares, rectangles (inner and outer rectangles), triangles or trapezia
  - consider functions which cannot be integrated in the scope of this syllabus, for example $f(x) = \ln x$, and explore the effect of increasing the number of shapes used

Students:

- use the notation of the definite integral $\int_a^b f(x)\,dx$ for the area under the curve $y = f(x)$ from $x = a$ to $x = b$ if $f(x) \geq 0$

Students:

- use the Trapezoidal rule to estimate areas under curves **AAM**
  - use geometric arguments (rather than substitution into a given formula) to approximate a definite integral of the form $\int_a^b f(x)\,dx$, where $f(x) \geq 0$, on the interval $a \leq x \leq b$, by dividing the area into a given number of trapezia with equal widths
  - demonstrate understanding of the formula: $\int_a^b f(x)\,dx \approx \frac{b-a}{2n}[f(a) + f(b) + 2\{f(x_1) + \cdots + f(x_{n-1})\}]$ where $a = x_0$ and $b = x_n$, and the values of $x_0, x_1, x_2, \ldots, x_n$ are found by dividing the interval $a \leq x \leq b$ into n equal sub-intervals

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Students:

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Students:

- use geometric ideas to find the definite integral $\int_a^b f(x)\,dx$ where $f(x)$ is positive throughout an interval $a \leq x \leq b$ and the shape of $f(x)$ allows such calculations, for example when $f(x)$ is a straight line in the interval or $f(x)$ is a semicircle in the interval **AAM**

Students:

- understand the relationship of position to signed areas, namely that the signed area above the horizontal axis is positive and the signed area below the horizontal axis is negative

Students:

- using technology or otherwise, investigate the link between the anti-derivative and the area under a curve
  - interpret $\int_a^b f(x)\,dx$ as a sum of signed areas (ACMMM127)
  - understand the concept of the signed area function $F(x) = \int_a^x f(t)\,dt$ (ACMMM129)

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Students:

- use the formula $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is the anti-derivative of $f(x)$, to calculate definite integrals (ACMMM131) **AAM**
  - understand and use the Fundamental Theorem of Calculus, $F'(x) = \frac{d}{dx}[\int_a^x f(t)\,dt] = f(x)$ and illustrate its proof geometrically (ACMMM130)
  - use symmetry properties of even and odd functions to simplify calculations of area
  - recognise and use the additivity and linearity of definite integrals (ACMMM128)
  - calculate total change by integrating instantaneous rate of change

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Students:

- calculate the area under a curve (ACMMM132)

Students:

- calculate areas between curves determined by any functions within the scope of this syllabus (ACMMM134) **AAM**

Students:

- integrate functions and find indefinite or definite integrals and apply this technique to solving practical problems **AAM**

Students:

Topic: Financial Mathematics

Outcomes

A student:

- models and solves problems and makes informed decisions about financial situations using mathematical reasoning and techniques MA12-2
- applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems MA12-4
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Topic Focus

The topic Financial Mathematics involves sequences and series and their application to financial situations.

A knowledge of financial mathematics enables analysis and interpretation of different financial situations, the calculation of the best options for the circumstances, and the solving of financial problems.

The study of financial mathematics is important in developing students' ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources prudently.

Subtopics

MA-M1 Modelling Financial Situations 📎

Financial Mathematics

MA-M1 Modelling Financial Situations

Outcomes

A student:

- models and solves problems and makes informed decisions about financial situations using mathematical reasoning and techniques MA12-2
- applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems MA12-4
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is the meaning and mathematics of annuities, including the introduction of arithmetic and geometric sequences and series with their application to financial situations.

Students develop an understanding for the use of series in the borrowing and investing of money, which are common activities for many adults in contemporary society. Annuities represent financial plans involving the sum of a geometric series and can be used to model regular savings plans, including superannuation.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

Content

M1.1: Modelling investments and loans

Students:

- solve compound interest problems involving financial decisions, including but not limited to a home loan, a savings account, a car loan or superannuation **AAM**
  - identify an annuity (present or future value) as an investment account with regular, equal contributions and interest compounding at the end of each period, or a single-sum investment from which regular, equal withdrawals are made
  - use technology to model an annuity as a recurrence relation and investigate (numerically or graphically) the effect of varying the interest rate or the amount and frequency of each contribution or a withdrawal on the duration and/or future or present value of the annuity
  - use a table of future value interest factors to perform annuity calculations, eg calculating the future value of an annuity, the contribution amount required to achieve a given future value or the single sum that would produce the same future value as a given annuity

Students:

M1.2: Arithmetic sequences and series

Students:

- know the difference between a sequence and a series

Students:

- recognise and use the recursive definition of an arithmetic sequence: $T_n = T_{n-1} + d,\ T_1 = a$ **AAM**

Students:

- establish and use the formula for the n^{th} term (where n is a positive integer) of an arithmetic sequence: $T_n = a + (n-1)\,d$, where a is the first term and d is the common difference, and recognise its linear nature **AAM**

Students:

- establish and use the formulae for the sum of the first n terms of an arithmetic sequence: $S_n = \frac{n}{2}(a + l)$ where l is the last term in the sequence and $S_n = \frac{n}{2}\{2a + (n-1)\,d\}$ **AAM**

Students:

- identify and use arithmetic sequences and arithmetic series in contexts involving discrete linear growth or decay such as simple interest (ACMMM070) **AAM**

M1.3: Geometric sequences and series

Students:

- recognise and use the recursive definition of a geometric sequence: $T_n = rT_{n-1},\ T_1 = a$ (ACMMM072) **AAM**

Students:

- establish and use the formula for the n^{th} term of a geometric sequence: $T_n = ar^{n-1}$, where a is the first term, r is the common ratio and n is a positive integer, and recognise its exponential nature (ACMMM073) **AAM**

Students:

- establish and use the formula for the sum of the first n terms of a geometric sequence: $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$ (ACMMM075) **AAM**

Students:

- derive and use the formula for the limiting sum of a geometric series with $|r| < 1$: $S = \frac{a}{1-r}$ **AAM**
  - understand the limiting behaviour as $n \to \infty$ and its application to a geometric series as a limiting sum
  - use the notation $\lim_{n \to \infty} r^n = 0$ for $|r| < 1$

Students:

M1.4: Financial applications of sequences and series

Students:

- use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMMM076) **AAM**
  - calculate the effective annual rate of interest and use results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
  - solve problems involving compound interest loans or investments, eg determining the future value of an investment or loan, the number of compounding periods for an investment to exceed a given value and/or the interest rate needed for an investment to exceed a given value (ACMGM096)
  - recognise a reducing balance loan as a compound interest loan with periodic repayments, and solve problems including the amount owing on a reducing balance loan after each payment is made

Students:

Students:

- solve problems involving financial decisions, including but not limited to a home loan, a savings account, a car loan or superannuation **AAM**
  - calculate the future value or present value of an annuity by developing an expression for the sum of the calculated compounded values of each contribution and using the formula for the sum of the first n terms of a geometric sequence
  - verify entries in tables of future values or annuities by using geometric series

Students:

Topic: Statistical Analysis

Outcomes

A student:

- › solves problems using appropriate statistical processes MA12-8
- › chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- › constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Topic Focus

The topic Statistical Analysis involves the exploration, display, analysis and interpretation of data to identify and communicate key information.

Knowledge of statistical analysis enables careful interpretation of situations and an awareness of the contributing factors when presented with information by third parties, including its possible misrepresentation.

The study of statistical analysis is important in developing students' ability to recognise, describe and apply statistical techniques in order to analyse current situations or to predict future outcomes. It also develops an awareness of how conclusions drawn from data can be used to inform decisions made by groups such as scientific investigators, business people and policy-makers.

Subtopics

MA-S2 Descriptive Statistics and Bivariate Data Analysis
MA-S3 Random Variables

Statistical Analysis

MA-S2 Descriptive Statistics and Bivariate Data Analysis

Outcomes

A student:

- solves problems using appropriate statistical processes MA12-8
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to introduce students to some methods for identifying, analysing and describing associations between pairs of variables (bivariate data).

Students develop the ability to display, interpret and analyse statistical relationships within bivariate data. Statistical results form the basis of many decisions affecting society, and also inform individual decision-making.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

Content

S2.1: Data (grouped and ungrouped) and summary statistics

Students:

- classify data relating to a single random variable

Students:

- organise, interpret and display data into appropriate tabular and/or graphical representations including but not limited to Pareto charts, cumulative frequency distribution tables or graphs, parallel box-plots and two-way tables **AAM**
  - compare the suitability of different methods of data presentation in real-world contexts (ACMEM048)

Students:

- summarise and interpret grouped and ungrouped data through appropriate graphs and summary statistics **AAM**

Students:

- calculate measures of central tendency and spread and investigate their suitability in real-world contexts and use to compare large datasets
  - investigate real-world examples from the media illustrating appropriate and inappropriate uses or misuses of measures of central tendency and spread (ACMEM056) **AAM**

Students:

- identify outliers and investigate and describe the effect of outliers on summary statistics
  - use different approaches for identifying outliers, for example consideration of the distance from the mean or median, or the use of below $Q_1 - 1.5 \times IQR$ and above $Q_3 + 1.5 \times IQR$ as criteria, recognising and justifying when each approach is appropriate
  - investigate and recognise the effect of outliers on the mean, median and standard deviation

Students:

- describe, compare and interpret the distributions of graphical displays and/or numerical datasets and report findings in a systematic and concise manner **AAM**

S2.2: Bivariate data analysis

Students:

- construct a bivariate scatterplot to identify patterns in the data that suggest the presence of an association (ACMGM052)

Students:

- use bivariate scatterplots (constructing them where needed), to describe the patterns, features and associations of bivariate datasets, justifying any conclusions **AAM**
  - describe bivariate datasets in terms of form (linear/non-linear) and in the case of linear, also the direction (positive/negative) and strength of association (strong/moderate/weak)
  - identify the dependent and independent variables within bivariate datasets where appropriate
  - describe and interpret a variety of bivariate datasets involving two numerical variables using real-world examples in the media or those freely available from government or business datasets

Students:

- calculate and interpret Pearson's correlation coefficient (r) using technology to quantify the strength of a linear association of a sample (ACMGM054)

Students:

- model a linear relationship by fitting an appropriate line of best fit to a scatterplot and using it to describe and quantify associations **AAM**
  - fit a line of best fit to the data by eye and using technology (ACMEM141, ACMEM142)
  - fit a least-squares regression line to the data using technology (ACMGM057)
  - interpret the intercept and gradient of the fitted line (ACMGM059)

Students:

- use the appropriate line of best fit, both found by eye and by applying the equation of the fitted line, to make predictions by either interpolation or extrapolation **AAM**
  - distinguish between interpolation and extrapolation, recognising the limitations of using the fitted line to make predictions, and interpolate from plotted data to make predictions where appropriate

Students:

- implement the statistical investigation process to answer questions by identifying, analysing and describing associations between two numeric variables **AAM**

Students:

- construct, interpret and analyse scatterplots for bivariate numerical data in practical contexts **AAM**
  - demonstrate an awareness of issues of privacy and bias, ethics, and responsiveness to diverse groups and cultures when collecting and using data

Students:

Statistical Analysis

MA-S3 Random Variables

Outcomes

A student:

- › solves problems using appropriate statistical processes MA12-8
- › chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- › constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Subtopic Focus

The principal focus of this subtopic is to introduce students to continuous random variables, the normal distribution and its use in a variety of contexts.

Students develop understanding of the probability density function, how integration or the area under the function determines probabilities to solve problems involving random variables, and an understanding of the normal distribution, its properties and uses. Students make connections between calculus skills developed earlier in the course and their applications in Statistics, and lay the foundations for future study in this area.

Content

S3.1: Continuous random variables

Students:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)

Students:

- understand and use the concepts of a probability density function of a continuous random variable **AAM**
  - know the two properties of a probability density function: $f(x) \geq 0$ for all real x and $\int_{-\infty}^{\infty} f(x)dx = 1$
  - define the probability as the area under the graph of the probability density function using the notation $P(X \leq x) = \int_a^x f(x)dx$, where $f(x)$ is the probability density function defined on $[a, b]$
  - examine simple types of continuous random variables and use them in appropriate contexts
  - explore properties of a random variable that is uniformly distributed
  - find the mode from a given probability density function

Students:

- obtain and analyse a cumulative distribution function with respect to a given probability density function
  - understand the meaning of a cumulative distribution function with respect to a given probability density function
  - use a cumulative distribution function to calculate the median and other percentiles

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S3.2: The normal distribution

Students:

- identify the numerical and graphical properties of data that is normally distributed 📎

Students:

- calculate probabilities and quantiles associated with a given normal distribution using technology and otherwise, and use these to solve practical problems (ACMMM170) **AAM**
  - identify contexts that are suitable for modelling by normal random variables, eg the height of a group of students (ACMMM168)
  - recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ, and the use of the standard normal distribution (ACMMM169)
  - visually represent probabilities by shading areas under the normal curve, eg identifying the value above which the top 10% of data lies

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Students:

- understand and calculate the z-score (standardised score) corresponding to a particular value in a dataset **AAM**
  - use the formula $z = \frac{x-\mu}{\sigma}$, where μ is the mean and σ is the standard deviation
  - describe the z-score as the number of standard deviations a value lies above or below the mean

Students:

- use z-scores to compare scores from different datasets, for example comparing students' subject examination scores **AAM**

Students:

- use collected data to illustrate the empirical rules for normally distributed random variables
  - apply the empirical rule to a variety of problems
  - sketch the graphs of $f(x) = e^{-x^2}$ and the probability density function for the normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ using technology
  - verify, using the Trapezoidal rule, the results concerning the areas under the normal curve

Students:

- use z-scores to identify probabilities of events less or more extreme than a given event **AAM**

Students:

- use z-scores to make judgements related to outcomes of a given event or sets of data **AAM**