Year 11 Extension 1 Maths

Calculus n noun
1 (plural calculuses) (also infinitesimal calculus) the branch of mathematics concerned with the determination and properties of derivatives and integrals of functions, by methods based on the summation of infinitesimal differences.
a particular method or system of calculation or reasoning.

ORIGIN
C17: from Latin, literally 'small pebble'.

"The mind that opens to a new idea never returns to its original size."
Albert Einstein

3.14159265...

Topic: Functions

Outcomes

A student:

- uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- manipulates algebraic expressions and graphical functions to solve problems ME11-2
- uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Topic Focus

The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities. This topic provides the means to more fully understand the behaviour of functions, extending to include inequalities, absolute values and inverse functions.

A knowledge of functions enables students to discover connections between algebraic and graphical representations, to determine solutions of equations and to model theoretical or real-life situations involving algebra.

The study of functions is important in developing students' ability to find, recognise and use connections, to communicate concisely and precisely, to use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes in areas such as finance, economics and weather.

Subtopics

ME-F1 Further Work with Functions
ME-F2 Polynomials

Functions

ME-F1 Further Work with Functions

Outcomes

A student:

- › uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- › manipulates algebraic expressions and graphical functions to solve problems ME11-2
- › uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- › communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Subtopic Focus

The principal focus of this subtopic is to further explore functions in a variety of contexts including: reciprocal and inverse functions, manipulating graphs of functions, and parametric representation of functions. The study of inequalities is an application of functions and enables students to express domains and ranges as inequalities.

Students develop proficiency in methods to identify solutions to equations both algebraically and graphically. The study of inverse functions is important in higher Mathematics and the calculus of these is studied later in the course. The study of parameters sets foundations for later work on projectiles.

Content

F1.1: Graphical relationships

Students:

- examine the relationship between the graph of $y = f(x)$ and the graph of $y = \frac{1}{f(x)}$ and hence sketch the graphs (ACMSM099)

Students:

- examine the relationship between the graph of $y = f(x)$ and the graphs of $y^2 = f(x)$ and $y = \sqrt{f(x)}$ and hence sketch the graphs

Students:

- examine the relationship between the graph of $y = f(x)$ and the graphs of $y = |f(x)|$ and $y = f(|x|)$ and hence sketch the graphs (ACMSM099)

Students:

- examine the relationship between the graphs of $y = f(x)$ and $y = g(x)$ and the graphs of $y = f(x) + g(x)$ and $y = f(x)g(x)$ and hence sketch the graphs

Students:

- apply knowledge of graphical relationships to solve problems in practical and abstract contexts **AAM**

F1.2: Inequalities

Students:

- solve quadratic inequalities using both algebraic and graphical techniques

Students:

- solve inequalities involving rational expressions, including those with the unknown in the denominator

Students:

- solve absolute value inequalities of the form $|ax + b| \geq k$, $|ax + b| \leq k$, $|ax + b| < k$ and $|ax + b| > k$

F1.3: Inverse functions

Students:

- define the inverse relation of a function $y = f(x)$ to be the relation obtained by reversing all the ordered pairs of the function

Students:

- examine and use the reflection property of the graph of a function and the graph of its inverse (ACMSM096)
  - understand why the graph of the inverse relation is obtained by reflecting the graph of the function in the line $y = x$
  - using the fact that this reflection exchanges horizontal and vertical lines, recognise that the horizontal line test can be used to determine whether the inverse relation of a function is again a function

Students:

- write the rule or rules for the inverse relation by exchanging x and y in the function rules, including any restrictions, and solve for y, if possible

---

Students:

- when the inverse relation is a function, use the notation $f^{-1}(x)$ and identify the relationships between the domains and ranges of $f(x)$ and $f^{-1}(x)$

Students:

- when the inverse relation is not a function, restrict the domain to obtain new functions that are one-to-one, and compare the effectiveness of different restrictions

---

Students:

- solve problems based on the relationship between a function and its inverse function using algebraic or graphical techniques **AAM**

F1.4: Parametric form of a function or relation

Students:

- understand the concept of parametric representation and examine lines, parabolas and circles expressed in parametric form
  - understand that linear and quadratic functions, and circles can be expressed in either parametric form or Cartesian form
  - convert linear and quadratic functions, and circles from parametric form to Cartesian form and vice versa
  - sketch linear and quadratic functions, and circles expressed in parametric form

Students:

Functions

ME-F2 Polynomials

Outcomes

A student:

- uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- manipulates algebraic expressions and graphical functions to solve problems ME11-2
- uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Subtopic Focus

The principal focus of this subtopic is to explore the behaviour of polynomials algebraically, including the remainder and factor theorems, and sums and products of roots.

Students develop knowledge, skills and understanding to manipulate, analyse and solve polynomial equations. Polynomials are of fundamental importance in algebra and have many applications in higher mathematics. They are also significant in many other fields of study, including the sciences, engineering, finance and economics.

Content

F2.1: Remainder and factor theorems

Students:

- define a general polynomial in one variable, x, of degree n with real coefficients to be the expression: $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, where $a_n \neq 0$
  - understand and use terminology relating to polynomials including degree, leading term, leading coefficient and constant term

---

Students:

- use division of polynomials to express $P(x)$ in the form $P(x) = A(x).Q(x) + R(x)$ where $\deg R(x) < \deg A(x)$ and $A(x)$ is a linear or quadratic divisor, $Q(x)$ the quotient and $R(x)$ the remainder
  - review the process of division with remainders for integers
  - describe the process of division using the terms: dividend, divisor, quotient, remainder

Students:

- prove and apply the factor theorem and the remainder theorem for polynomials and hence solve simple polynomial equations (ACMSM089, ACMSM091)

F2.2: Sums and products of roots of polynomials

Students:

- solve problems using the relationships between the roots and coefficients of quadratic, cubic and quartic equations **AAM**
  - consider quadratic, cubic and quartic equations, and derive formulae as appropriate for the sums and products of roots in terms of the coefficients

Students:

- determine the multiplicity of a root of a polynomial equation
  - prove that if a polynomial equation of the form $P(x) = 0$ has a root of multiplicity $r > 1$, then $P'(x) = 0$ has a root of multiplicity $r - 1$

....

....

....

....

....

....

....

....

....

....

....

....

....

Students:

- graph a variety of polynomials and investigate the link between the root of a polynomial equation and the zero on the graph of the related polynomial function
  - examine the sign change of the function and shape of the graph either side of roots of varying multiplicity

....

....

....

....

....

....

....

....

....

....

....

....

Topic: Trigonometric Functions

Outcomes

A student:

- › uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- › applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems ME11-3
- › uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- › communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Topic Focus

The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations. It extends to exploration and understanding of inverse trigonometric functions over restricted domains and their behaviour in both algebraic and graphical form.

A knowledge of trigonometric functions enables the solving of problems involving inverse trigonometric functions, and the modelling of the behaviour of naturally occurring periodic phenomena such as waves and signals to solve problems and to predict future outcomes.

The study of the graphs of trigonometric functions is important in developing students' understanding of the connections between algebraic and graphical representations and how this can be applied to solve problems from theoretical or real-life scenarios and situations.

Subtopics

ME-T1 Inverse Trigonometric Functions
ME-T2 Further Trigonometric Identities

Trigonometric Functions

ME-T1 Inverse Trigonometric Functions

Outcomes

A student:

- › uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- › applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems ME11-3
- › uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- › communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Subtopic Focus

The principal focus of this subtopic is for students to determine and to work with the inverse trigonometric functions.

Students explore inverse trigonometric functions which are important examples of inverse functions. They sketch the graphs of these functions and apply a range of properties to extend their knowledge and understanding of the connections between algebraic and geometrical representations of functions. This enables a deeper understanding of the nature of periodic functions, which are used as powerful modelling tools for any quantity that varies in a cyclical way.

Content

Students:

- define and use the inverse trigonometric functions (ACMSM119)
  - understand and use the notation $\arcsin x$ and $\sin^{-1}x$ for the inverse function of $\sin x$ when $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (and similarly for $\cos x$ and $\tan x$) and understand when each notation might be appropriate to avoid confusion with the reciprocal functions
  - use the convention of restricting the domain of $\sin x$ to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, so the inverse function exists. The inverse of this restricted sine function is defined by: $y = \sin^{-1}x$, $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
  - use the convention of restricting the domain of $\cos x$ to $0 \leq x \leq \pi$, so the inverse function exists. The inverse of this restricted cosine function is defined by: $y = \cos^{-1}x,\ -1 \leq x \leq 1$ and $0 \leq y \leq \pi$
  - use the convention of restricting the domain of $\tan x$ to $-\frac{\pi}{2} < x < \frac{\pi}{2}$, so the inverse function exists. The inverse of this restricted tangent function is defined by: $y = \tan^{-1}x$, x is a real number and $-\frac{\pi}{2} < y < \frac{\pi}{2}$
  - classify inverse trigonometric functions as odd, even or neither odd nor even

Students:

Students:

- sketch graphs of the inverse trigonometric functions

Students:

- use the relationships $\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin x) = x$, $\cos(\cos^{-1} x) = x$ and $\cos^{-1}(\cos x) = x$, and $\tan(\tan^{-1} x) = x$ and $\tan^{-1}(\tan x) = x$ where appropriate, and state the values of x for which these relationships are valid

Students:

- prove and use the properties: $\sin^{-1}(-x) = -\sin^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $\tan^{-1}(-x) = -\tan^{-1}x$ and $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

....

....

....

....

....

....

....

....

....

....

....

....

....

Students:

- solve problems involving inverse trigonometric functions in a variety of abstract and practical situations **AAM**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Trigonometric Functions

ME-T2 Further Trigonometric Identities

Outcomes

A student:

- uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems ME11-3
- uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Subtopic Focus

The principal focus of this subtopic is for students to define and work with trigonometric identities to both prove results and manipulate expressions.

Students develop knowledge of how to manipulate trigonometric expressions to solve equations and to prove results. Trigonometric expressions and equations provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources, and alternating current. The solution of trigonometric equations may require the use of trigonometric identities.

Content

Students:

- derive and use the sum and difference expansions for the trigonometric functions $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ (ACMSM044)
  - $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  - $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
  - $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Students:

- derive and use the double angle formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ (ACMSM044)
  - $\sin 2A = 2 \sin A \cos A$
  - $\cos 2A = \cos^2 A - \sin^2 A$
    $= 2\cos^2 A - 1$
    $= 1 - 2\sin^2 A$
  - $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Students:

- derive and use expressions for $\sin A$, $\cos A$ and $\tan A$ in terms of t where $t = \tan\frac{A}{2}$ (the t-formulae)
  - $\sin A = \frac{2t}{1+t^2}$
  - $\cos A = \frac{1-t^2}{1+t^2}$
  - $\tan A = \frac{2t}{1-t^2}$

Students:

- derive and use the formulae for trigonometric products as sums and differences for $\cos A \cos B$, $\sin A \sin B$, $\sin A \cos B$ and $\cos A \sin B$ (ACMSM047)
  - $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
  - $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
  - $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$
  - $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

Topic: Calculus

Outcomes

A student:

- uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
- applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change ME11-4
- uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Topic Focus

The topic Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It involves the development of the connections between rates of change and related rates of change, the derivatives of functions and the manipulative skills necessary for the effective use of differential calculus.

The study of calculus is important in developing students' knowledge and understanding of related rates of change and developing the capacity to operate with and model situations involving change, using algebraic and graphical techniques to describe and solve problems and to predict outcomes with relevance to, for example the physical, natural and medical sciences, commerce and the construction industry.

Subtopics

ME-C1 Rates of Change

Calculus

ME-C1 Rates of Change

Outcomes

A student:

› uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
› applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change ME11-4
› uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
› communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Subtopic Focus

The principal focus of this subtopic is for students to solve problems involving the chain rule and differentiation of the exponential function, and understand how these concepts can be applied to the physical and natural sciences.

Students develop the ability to study motion problems in an abstract situation, which may in later studies be applied to large and small mechanical systems, from aeroplanes and satellites to miniature robotics. Students also study the mathematics of exponential growth and decay, two fundamental processes in the natural environment.

Content

C1.1: Rates of change with respect to time

Students:

- describe the rate of change of a physical quantity with respect to time as a derivative
  - investigate examples where the rate of change of some aspect of a given object with respect to time can be modelled using derivatives **AAM**
  - use appropriate language to describe rates of change, for example 'at rest', 'initially', 'change of direction' and 'increasing at an increasing rate'

Students:

- find and interpret the derivative $\frac{dQ}{dt}$, given a function in the form $Q = f(t)$, for the amount of a physical quantity present at time t

Students:

- describe the rate of change with respect to time of the displacement of a particle moving along the x-axis as a derivative $\frac{dx}{dt}$ or $\dot{x}$

Students:

- describe the rate of change with respect to time of the velocity of a particle moving along the x-axis as a derivative $\frac{d^2x}{dt^2}$ or $\ddot{x}$

C1.2: Exponential growth and decay

Students:

- construct, analyse and manipulate an exponential model of the form $N(t) = Ae^{kt}$ to solve a practical growth or decay problem in various contexts (for example population growth, radioactive decay or depreciation) **AAM**
  - establish the simple growth model, $\frac{dN}{dt} = kN$, where N is the size of the physical quantity, $N = N(t)$ at time t and k is the growth constant
  - verify (by substitution) that the function $N(t) = Ae^{kt}$ satisfies the relationship $\frac{dN}{dt} = kN$, with A being the initial value of N
  - sketch the curve $N(t) = Ae^{kt}$ for positive and negative values of k
  - recognise that this model states that the rate of change of a quantity varies directly with the size of the quantity at any instant

Students:

Students:

- establish the modified exponential model, $\frac{dN}{dt} = k(N - P)$, for dealing with problems such as 'Newton's Law of Cooling' or an ecosystem with a natural 'carrying capacity' **AAM**
  - verify (by substitution) that a solution to the differential equation $\frac{dN}{dt} = k(N - P)$ is $N(t) = P + Ae^{kt}$, for an arbitrary constant A, and P a fixed quantity, and that the solution is $N = P$ in the case when $A = 0$
  - sketch the curve $N(t) = P + Ae^{kt}$ for positive and negative values of k
  - note that whenever $k < 0$, the quantity N tends to the limit P as $t \to \infty$, irrespective of the initial conditions
  - recognise that this model states that the rate of change of a quantity varies directly with the difference in the size of the quantity and a fixed quantity at any instant

....

....

....

....

....

....

....

....

Students:

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Students:

- solve problems involving situations that can be modelled using the exponential model or the modified exponential model and sketch graphs appropriate to such problems **AAM**

C1.3: Related rates of change

Students:

- solve problems involving related rates of change as instances of the chain rule (ACMSM129) **AAM**

Students:

- develop models of contexts where a rate of change of a function can be expressed as a rate of change of a composition of two functions, and to which the chain rule can be applied

Students:

Topic: Combinatorics

Outcomes

A student:

- uses concepts of permutations and combinations to solve problems involving counting or ordering ME11-5
- uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Topic Focus

The topic Combinatorics involves counting and ordering as well as exploring arrangements, patterns, symmetry and other methods to generalise and predict outcomes. The consideration of the expansion of $(x + y)^n$, where n is a positive integer, draws together aspects of number theory and probability theory.

A knowledge of combinatorics is useful when considering situations and solving problems involving counting, sorting and arranging. Efficient counting methods have many applications and are used in the study of probability.

The study of combinatorics is important in developing students' ability to generalise situations, to explore patterns and to ensure the consideration of all outcomes in situations such as the placement of people or objects, setting-up of surveys, jury or committee selection and design.

Subtopics

ME-A1 Working with Combinatorics

Combinatorics

ME-A1 Working with Combinatorics

Outcomes

A student:

- › uses concepts of permutations and combinations to solve problems involving counting or ordering ME11-5
- › uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- › communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

Subtopic Focus

The principal focus of this subtopic is to develop students' understanding and proficiency with permutations and combinations and their relevance to the binomial coefficients.

Students develop proficiency in ordering and counting techniques in both restricted and unrestricted situations. The binomial expansion is introduced, Pascal's triangle is constructed and related identities are proved. The material studied provides the basis for more advanced work, where the binomial expansion is extended to cases for rational values of n, and applications in calculus are explored.

Content

A1.1: Permutations and combinations

Students:

- list and count the number of ways an event can occur

Students:

- use the fundamental counting principle (also known as the multiplication principle)

Students:

- use factorial notation to describe and determine the number of ways n different items can be arranged in a line or a circle
  - solve problems involving cases where some items are not distinct (excluding arrangements in a circle)

....

....

....

....

....

....

....

....

....

....

....

....

Students:

- solve simple problems and prove results using the pigeonhole principle (ACMSM006)
  - understand that if there are n pigeonholes and $n+1$ pigeons to go into them, then at least one pigeonhole must hold 2 or more pigeons
  - generalise to: If n pigeons are sitting in k pigeonholes, where $n > k$, then there is at least one pigeonhole with at least $\frac{n}{k}$ pigeons in it
  - prove the pigeonhole principle

....

....

....

....

....

....

....

....

....

....

....

Students:

- understand and use permutations to solve problems (ACMSM001)
  - understand and use the notation ^nP_r and the formula $^nP_r = \frac{n!}{(n-r)!}$

Students:

- solve problems involving permutations and restrictions with or without repeated objects (ACMSM004)

Students:

- understand and use combinations to solve problems (ACMSM007)
  - understand and use the notations $\binom{n}{r}$ and ${}^{n}C_{r}$ and the formula ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ (ACMMM045, ACMSM008)

Students:

- solve practical problems involving permutations and combinations, including those involving simple probability situations **AAM**

A1.2: The binomial expansion and Pascal's triangle

Students:

- expand $(x + y)^n$ for small positive integers n (ACMMM046)
  - note the pattern formed by the coefficients of x in the expansion of $(1 + x)^n$ and recognise links to Pascal's triangle
  - recognise the numbers $\binom{n}{r}$ (also denoted ${}^nC_r$) as binomial coefficients (ACMMM047)

....

....

....

....

....

....

....

....

....

....

....

Students:

- derive and use simple identities associated with Pascal's triangle (ACMSM009)
  - establish combinatorial proofs of the Pascal's triangle relations ${}^nC_0 = 1$, ${}^nC_n = 1$; ${}^nC_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$ for $1 \le k \le n - 1$; and ${}^nC_k = {}^nC_{n-k}$

....

....

....

....

....

....

....

....

....

....

....

....

....