Advanced Mathematics

Year 11

'Tis a favorite project of mine;

A new value of pi to assign.

I would fix it at 3;

For it's simpler, you see,

Than 3 point 1 4 1 5 9.

The mind is not a
vessel to be filled,
but a fire to be kindled.
Plutarch

Topic: Functions

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› uses the concepts of functions and relations to model, analyse and solve practical problems MA11-2
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Topic Focus

The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities.

A knowledge of functions enables students to discover, recognise and generalise connections between algebraic and graphical representations of the same expression and to describe interactions through the use of both dependent and independent variables.

The study of functions is important in developing students' ability to find connections and patterns, to communicate concisely and precisely, to use algebraic techniques and manipulations, to describe and solve problems, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather.

Subtopics

MA-F1 Working with Functions 📎

Functions

MA-F1 Working with Functions 📎

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› uses the concepts of functions and relations to model, analyse and solve practical problems MA11-2
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Subtopic Focus

The principal focus of this subtopic is to introduce students to the concept of a function and develop their knowledge of functions and their respective graphs. Function notation is introduced, which is essential for describing the ideas of calculus.

Students develop their use of mathematical language to describe functions, their properties and respective graphs while applying this knowledge to everyday problems and applications. In business and economics, for example revenue depends on the number of items sold, and expressing this relationship as a function allows the investigation of changes in revenue as sales change.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

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Content

F1.1: Algebraic techniques

Students:

- use index laws and surds

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Students:

- solve quadratic equations using the quadratic formula and by completing the square (ACMMM008)

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Students:
- manipulate complex algebraic expressions involving algebraic fractions

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F1.2: Introduction to functions

Students:

- define and use a function and a relation as mappings between sets, and as a rule or a formula that defines one variable quantity in terms of another
    - define a relation as any set of ordered pairs (x, y) of real numbers
    - understand the formal definition of a function as a set of ordered pairs (x, y) of real numbers such that no two ordered pairs have the same first component (or x-component)

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Students:

- use function notation, domain and range, independent and dependent variables (ACMMM023) ☞
  - understand and use interval notation as a way of representing domain and range

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Students:
- understand the concept of the graph of a function (ACMMM024)

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Students:

- identify types of functions and relations on a given domain, using a variety of methods
  – know what is meant by one-to-one, one-to-many, many-to-one and many-to-many
  – use the vertical line test to identify a function
  – determine if a function is one-to-one (ACMSM094)

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Students:

- define odd and even functions algebraically and recognise their geometric properties

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Students:
- define the sum, difference, product and quotient of functions and consider their domains and ranges where possible

Students:
- define and use the composite function $f(g(x))$ of functions $f(x)$ and $g(x)$ where appropriate
  - identify the domain and range of a composite function

Students:
- recognise that solving the equation $f(x) = 0$ corresponds to finding the values of x for which the graph of $y = f(x)$ cuts the x-axis (the x-intercepts)

Students:

F1.3: Linear, quadratic and cubic functions

Students:

- model, analyse and solve problems involving linear functions **AAM** 📎 ⚡ ⚙
    - recognise that a direct variation relationship produces a straight-line graph
    - explain the geometrical significance of m and c in the equation $f(x) = mx + c$
    - derive the equation of a straight line passing through a fixed point (x_1, y_1) and having a given gradient m using the formula $y - y_1 = m(x - x_1)$
    - derive the equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) by first calculating its gradient m using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
    - understand and use the fact that parallel lines have the same gradient and that two lines with gradient m_1 and m_2 respectively are perpendicular if and only if $m_1 m_2 = -1$
    - find the equations of straight lines, including parallel and perpendicular lines, given sufficient information (ACMMM004) ⚙

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Students:

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Students:

- model, analyse and solve problems involving quadratic functions **AAM** 📎🖥️💬🌾
  - recognise features of the graph of a quadratic, including its parabolic nature, turning point, axis of symmetry and intercepts (ACMMM007)
  - find the vertex and intercepts of a quadratic graph by either factorising, completing the square or solving the quadratic equation as appropriate
  - understand the role of the discriminant in relation to the position of the graph ⚙️
  - find the equation of a quadratic given sufficient information (ACMMM009)

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Students:

- solve practical problems involving a pair of simultaneous linear and/or quadratic functions algebraically and graphically, with or without the aid of technology; for example determining and interpreting the break-even point of a simple business problem **AAM** 📎🖥️✳️
  - understand that solving $f(x) = k$ corresponds to finding the values of x for which the graph $y = f(x)$ cuts the line $y = k$

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Students:

- recognise cubic functions of the form: $f(x) = kx^3$, $f(x) = k(x - b)^3 + c$ and $f(x) = k(x - a)(x - b)(x - c)$, where a, b, c and k are constants, from their equation and/or graph and identify important features of the graph 🖥

F1.4: Further functions and relations

Students:

- define a real polynomial $P(x)$ as the expression $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ where $n = 0,1,2, \ldots$ and $a_0, a_1, a_2, \ldots, a_n$ are real numbers

Students:
- identify the coefficients and the degree of a polynomial (ACMMM015)

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Students:
- identify the shape and features of graphs of polynomial functions of any degree in factored form and sketch their graphs

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Students:
- recognise that functions of the form $f(x) = \frac{k}{x}$ represent inverse variation, identify the hyperbolic shape of their graphs and identify their asymptotes **AAM** 📎💻

Students:
- define the absolute value $|x|$ of a real number x as the distance of the number from the origin on a number line without regard to its sign

Students:
- use and apply the notation $|x|$ for the absolute value of the real number x and the graph of $y = |x|$ (ACMSM098)
  - recognise the shape and features of the graph of $y = |ax + b|$ and hence sketch the graph

Students:
- solve simple absolute value equations of the form $|ax + b| = k$ both algebraically and graphically

Students:
- given the graph of $y = f(x)$, sketch $y = -f(x)$ and $y = f(-x)$ and $y = -f(-x)$ using reflections in the x and y-axes

Students:
- recognise features of the graphs of $x^2 + y^2 = r^2$ and $(x - a)^2 + (y - b)^2 = r^2$, including their circular shapes, their centres and their radii (ACMMM020) ✱ 💻
  - derive the equation of a circle, centre the origin, by considering Pythagoras' theorem and recognise that a circle is not a function
  - transform equations of the form $x^2 + y^2 + ax + by + c = 0$ into the form $(x - a)^2 + (y - b)^2 = r^2$, by completing the square
  - sketch circles given their equations and find the equation of a circle from its graph
  - recognise that $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$ are functions, identify the semicircular shape of their graphs and sketch them

Topic: Trigonometric Functions

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes MA11-3
› uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities MA11-4
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Topic Focus

The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations.

A knowledge of trigonometric functions enables the solving of practical problems involving triangles or periodic graphs, such as waves and signals.

The study of trigonometric functions is important in developing students' understanding of periodic behaviour, a property not possessed by any previously studied functions. Utilising this property, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

Subtopics

MA-T1 Trigonometry and Measure of Angles 📎
MA-T2 Trigonometric Functions and Identities

Trigonometric Functions

MA-T1 Trigonometry and Measure of Angles 📎

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes MA11-3
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Subtopic Focus

The principal focus of this subtopic is to solve problems involving triangles using trigonometry, and to understand and use angular measure expressed in radians and degrees. This has practical and analytical applications in areas including surveying, navigation, meteorology, architecture, construction and electronics.

Students develop techniques to solve problems involving triangles, and then extend these ideas to include the exact ratios for angles, and also to the study of non-right-angled triangles. This introduces the need to define the trigonometric ratios for obtuse angles, which is followed by the establishment of trigonometric ratios of angles of any size. Radians are introduced as another measure in which angles of any size can be found. Radians are important for the study of the calculus of trigonometric functions in Year 12.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

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Content

T1.1 Trigonometry

Students:

- use the sine, cosine and tangent ratios to solve problems involving right-angled triangles where angles are measured in degrees, or degrees and minutes 📎

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Students:

- establish and use the sine rule, cosine rule and the area of a triangle formula for solving problems where angles are measured in degrees, or degrees and minutes **AAM** 📎💻

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Students:
- find angles and sides involving the ambiguous case of the sine rule
    - use technology and/or geometric construction to investigate the ambiguous case of the sine rule when finding an angle, and the condition for it to arise ⚙ 💻

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Students:
- solve problems involving the use of trigonometry in two and three dimensions **AAM** 📎
    - interpret information about a two or three-dimensional context given in diagrammatic or written form and construct diagrams where required

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Students:
- solve practical problems involving Pythagoras' theorem and the trigonometry of triangles, which may involve the ambiguous case, including but not limited to finding and using angles of elevation and depression and the use of true bearings and compass bearings in navigation **AAM** 📎 ⚙️

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T1.2 Radians

Students:

- understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using degrees (ACMMM029)
    - sketch the trigonometric functions in degrees for $0° \leq x \leq 360°$

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Students:
- define and use radian measure and understand its relationship with degree measure (ACMMM032) 📖
  - convert between the two measures, using the fact that $360° = 2\pi$ radians
  - recognise and use the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in both degrees and radians for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035)

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Students:
- understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using radians (ACMMM034)

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Students:
- solve problems involving trigonometric ratios of angles of any magnitude in both degrees and radians

Students:
- recognise the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ and sketch on extended domains in degrees and radians (ACMMM036)

Students:

- derive the formula for arc length, $l = r\theta$ and for the area of a sector of a circle, $A = \frac{1}{2}r^2\theta$ 👉

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Students:

- solve problems involving sector areas, arc lengths and combinations of either areas or lengths

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Trigonometric Functions

MA-T2 Trigonometric Functions and Identities

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities MA11-4
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Subtopic Focus

The principal focus of this subtopic is to use trigonometric identities and reciprocal relationships to simplify expressions, to prove equivalences and to solve equations.

Students develop their ability to prove identities, simplify expressions and solve trigonometric equations. Trigonometric expressions and equations provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources, and alternating current. The solution of trigonometric equations may require the use of trigonometric identities.

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Content

Students:

- define the reciprocal trigonometric functions, $y = \operatorname{cosec} x$, $y = \sec x$ and $y = \cot x$

  - $\operatorname{cosec} A = \dfrac{1}{\sin A}$, $\sin A \neq 0$

  - $\sec A = \dfrac{1}{\cos A}$, $\cos A \neq 0$

  - $\cot A = \dfrac{\cos A}{\sin A}$, $\sin A \neq 0$

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Students:

- sketch the graphs of reciprocal trigonometric functions in both radians and degrees

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Students:
- prove and apply the Pythagorean identities $\cos^2 x + \sin^2 x = 1$, $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$ (ACMSM046)
  - know the difference between an equation and an identity

Students:
- use $\tan x = \dfrac{\sin x}{\cos x}$ provided that $\cos x \neq 0$

Students:

- prove trigonometric identities

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Students:

- evaluate trigonometric expressions using angles of any magnitude and complementary angle results

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Students:
- simplify trigonometric expressions and solve trigonometric equations, including those that reduce to quadratic equations

Students:

Topic: Calculus

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems MA11-5
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Topic Focus

The topic Calculus is concerned with how quantities change and provides a framework for developing quantitative models of change and deducing their consequences. The topic involves the development of the basic concepts upon which differential calculus is built, namely the connection between the gradient of the tangent to a curve and the instantaneous rate of change of a function, rates of change and derivatives of functions and the manipulative skills necessary for the effective use of differential calculus.

The study of calculus is important in developing students' ability to solve problems involving algebraic and graphical representations of functions and rates of change of a function with relevance to all quantitative fields of study including physics, chemistry, medicine, engineering, computing, statistics, business, finance, economics and the construction industry.

Subtopics

MA-C1 Introduction to Differentiation

Calculus

MA-C1 Introduction to Differentiation

Outcomes

A student:

› uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1
› interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems MA11-5
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Subtopic Focus

The principal focus of this subtopic is for students to develop an understanding of the concept of a derivative as a function that defines the rate of change of a given function. This concept is reinforced numerically, by calculating difference quotients, geometrically, as gradients of secants and tangents, and algebraically. The derivatives of power functions are found and used to solve simple problems, including calculating gradients and equations of tangents and normals.

Students develop an understanding of derivatives as representations of rates of change. This process is of fundamental importance in Mathematics and has applications in all quantitative fields of study including physics, chemistry, medicine, engineering, computing, statistics, business, finance and economics.

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Content

C1.1: Gradients of tangents

Students:

- distinguish between continuous and discontinuous functions, identifying key elements which distinguish each type of function
    - sketch graphs of functions that are continuous and compare them with graphs of functions that have discontinuities
    - describe continuity informally, and identify continuous functions from their graphs

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Students:

- describe the gradient of a secant drawn through two nearby points on the graph of a continuous function as an approximation of the gradient of the tangent to the graph at those points, which improves in accuracy as the distance between the two points decreases

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Students:

- examine and use the relationship between the angle of inclination of a line or tangent, θ, with the positive x-axis, and the gradient, m, of that line or tangent, and establish that $\tan \theta = m$ **AAM**

C1.2: Difference quotients

Students:

- describe the behaviour of a function and its tangent at a point, using language including increasing, decreasing, constant, stationary, increasing at an increasing rate **AAM** ⚙ 🖳

Students:

- interpret and use the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of $f(x)$ or the gradient of a chord or secant of the graph $y = f(x)$ ✴ ☞

Students:

- interpret the meaning of the gradient of a function in a variety of contexts, for example on distance–time or velocity–time graphs ✴ ☞

C1.3: The derivative function and its graph

Students:

- examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit (ACMMM081)

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Students:

- interpret the derivative as the gradient of the tangent to the graph of $y = f(x)$ at a point x (ACMMM085)

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Students:

- estimate numerically the value of the derivative at a point, for simple power functions (ACMMM086) ⚙ 💻

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Students:

- define the derivative $f'(x)$ from first principles, as $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and use the notation for the derivative: $\frac{dy}{dx} = f'(x) = y'$, where $y = f(x)$

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Students:

- use first principles to find the derivative of simple polynomials, up to and including degree 3

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Students:

- understand the concept of the derivative as a function (ACMMM089)

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Students:
- sketch the derivative function (or gradient function) for a given graph of a function, without the use of algebraic techniques and in a variety of contexts including but not limited to motion in a straight line 🔧 💻 ✏️
  - establish that $f'(x) = 0$ at a stationary point, $f'(x) > 0$ when the function is increasing and $f'(x) < 0$ when it is decreasing, to form a framework for sketching the derivative function
  - identify families of curves with the same derivative function (ACMMM121)
  - use technology to plot functions and their gradient functions

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Students:
- interpret and use the derivative at a point as the instantaneous rate of change of a function at that point **AAM**
  - examine examples of variable rates of change of non-linear functions (ACMMM087)

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C1.4: Calculating with derivatives

Students:

- use the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for all real values of n ⚙️

Students:

- differentiate a constant multiple of a function and the sum or difference of two functions ⚙️ 💻

Students:
- understand and use the product, quotient and chain rules to differentiate functions of the form $f(x)g(x), \frac{f(x)}{g(x)}$ and $f(g(x))$ where $f(x)$ and $g(x)$ are functions
  - apply the product rule: If $h(x) = f(x)\,g(x)$ then $h'(x) = f(x)\,g'(x) + f'(x)\,g(x)$, or if u and v are both functions of x then $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
  - apply the quotient rule: If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x)-f(x)g'(x)}{g(x)^2}$, or if u and v are both functions of x then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
  - apply the chain rule: If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))\,g'(x)$, or if y is a function of u and u is a function of x then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

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Students:

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Students:

- calculate derivatives of power functions to solve problems, including but not limited to finding an instantaneous rate of change of a function in both real life and abstract situations **AAM**

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Students:

- use the derivative in a variety of contexts, including but not limited to finding the equation of a tangent or normal to a graph of a power function at a given point **AAM**

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Students:
- determine the velocity of a particle given its displacement from a point as a function of time

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Students:
- determine the acceleration of a particle given its velocity at a point as a function of time

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Topic: Exponential and Logarithmic Functions

Outcomes

A student:

› manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems MA11-6
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Topic Focus

The topic Exponential and Logarithmic Functions introduces exponential and logarithmic functions and develops their properties, including the manipulation of expressions involving them. The exponential function e^x is introduced by considering graphs of the derivative of exponential functions.

A knowledge of exponential and logarithmic functions enables an understanding of practical applications, such as exponential growth and decay, as well as applications within the Calculus topic.

The study of exponential and logarithmic functions is important in developing students' ability to solve practical problems involving rates of change in contexts such as population growth and compound interest.

Subtopics

MA-E1 Logarithms and Exponentials 📎

Exponential and Logarithmic Functions

MA-E1 Logarithms and Exponentials 📎

Outcomes

A student:

› manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems MA11-6
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Subtopic Focus

The principal focus of this subtopic is for students to learn about Euler's number e, become fluent in manipulating logarithms and exponentials and to use their knowledge, skills and understanding to solve problems relating to exponentials and logarithms.

Students develop an understanding of numbering systems, their representations and connections to observable phenomena such as exponential growth and decay. The exponential and logarithmic functions $f(x) = e^x$ and $f(x) = \log_e x$ are important non-linear functions in Mathematics, and have many applications in industry, finance and science. They are also fundamental functions in the study of more advanced Mathematics.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

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Content

E1.1: Introducing logarithms

Students:

- define logarithms as indices: $y = a^x$ is equivalent to $x = \log_a y$, and explain why this definition only makes sense when $a > 0$, $a \neq 1$

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Students:

- recognise and sketch the graphs of $y = a^x$, $y = a^{-x}$ and $y = \log_a x$ 📎

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Students:

- recognise and use the inverse relationship between logarithms and exponentials
  - understand and use the fact that $\log_a a^x = x$ for all real x, and $a^{\log_a x} = x$ for all $x > 0$

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Students:

E1.2: Logarithmic laws and applications

- derive the logarithmic laws from the index laws and use the algebraic properties of logarithms to simplify and evaluate logarithmic expressions

$\log_a m + \log_a n = \log_a(mn)$, $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$, $\log_a(m^n) = n\log_a m$,

$\log_a a = 1, \log_a 1 = 0, \log_a \frac{1}{x} = -\log_a x$

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Students:

- consider different number bases and prove and use the change of base law $\log_a x = \frac{\log_b x}{\log_b a}$ **AAM**

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Students:

- interpret and use logarithmic scales, for example decibels in acoustics, different seismic scales for earthquake magnitude, octaves in music or pH in chemistry (ACMMM154) **AAM**

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Students:

- solve algebraic, graphical and numerical problems involving logarithms in a variety of practical and abstract contexts, including applications from financial, scientific, medical and industrial contexts **AAM** ⚙

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Students:

E1.3: The exponential function and natural logarithms

- establish and use the formula $\frac{d(e^x)}{dx} = e^x$ (ACMMM100)
  - using technology, sketch and explore the gradient function of exponential functions and determine that there is a unique number $e \approx 2.71828182845$, for which $\frac{d(e^x)}{dx} = e^x$ where e is called Euler's number 💻

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Students:
- apply the differentiation rules to functions involving the exponential function, $f(x) = ke^{ax}$, where k and a are constants

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Students:
- work with natural logarithms in a variety of practical and abstract contexts **AAM**
  - define the natural logarithm $\ln x = \log_e x$ from the exponential function $f(x) = e^x$ (ACMMM159)
  - recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$ (ACMMM160)
  - use the natural logarithm and the relationships $e^{\ln x} = x$ where $x > 0$, and $\ln(e^x) = x$ for all real x in both algebraic and practical contexts
  - use the logarithmic laws to simplify and evaluate natural logarithmic expressions and solve equations

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Students:

E1.4: Graphs and applications of exponential and logarithmic functions

- solve equations involving indices using logarithms (ACMMM155)

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Students:
- graph an exponential function of the form $y = a^x$ for $a > 0$ and its transformations $y = ka^x + c$ and $y = ka^{x+b}$ where k, b and c are constants 📎🖥
  - interpret the meaning of the intercepts of an exponential graph and explain the circumstances in which these do not exist ✴

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Students:

- establish and use the algebraic properties of exponential functions to simplify and solve problems (ACMMM064)

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Students:

- solve problems involving exponential functions in a variety of practical and abstract contexts, using technology, and algebraically in simple cases (ACMMM067) **AAM** 📎💻

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Students:
- graph a logarithmic function $y = \log_a x$ for $a > 0$ and its transformations $y = k \log_a x + c$, using technology or otherwise, where k and c are constants 🖥
  - recognise that the graphs of $y = a^x$ and $y = \log_a x$ are reflections in the line $y = x$

Students:
- model situations and solve simple equations involving logarithmic or exponential functions algebraically and graphically **AAM**

Students:

- identify contexts suitable for modelling by exponential and logarithmic functions and use these functions to solve practical problems (ACMMM066, ACMMM158) **AAM**

Students:

Topic: Statistical Analysis

Outcomes

A student:

› uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-7
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Topic Focus

The topic Statistical Analysis involves the exploration, display, analysis and interpretation of data to identify and communicate key information.

A knowledge of statistical analysis enables careful interpretation of situations and an awareness of the contributing factors when presented with information by third parties, including its possible misrepresentation.

The study of statistical analysis is important in developing students' ability to recognise, describe and apply statistical techniques in order to analyse current situations or to predict future outcomes. It also develops an awareness of how conclusions drawn from data can be used to inform decisions made by groups such as scientific investigators, business people and policy-makers.

Subtopics

MA-S1 Probability and Discrete Probability Distributions 📎

Statistical Analysis

MA-S1 Probability and Discrete Probability Distributions 📎

Outcomes

A student:
› uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-7
› uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
› provides reasoning to support conclusions which are appropriate to the context MA11-9

Subtopic Focus

The principal focus of this subtopic is to introduce the concepts of conditional probability and independence and develop an understanding of discrete random variables and their uses in modelling random processes involving chance.

Students develop their skills related to probability, its language and visual representations, and use these skills to solve practical problems. They develop an understanding of probability distributions and associated statistical analysis methods and their use in modelling binomial events. These concepts play an important role in later studies of statistics, particularly in beginning to understand the concept of statistical significance.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

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Content

S1.1: Probability and Venn diagrams

Students:

- understand and use the concepts and language associated with theoretical probability, relative frequency and the probability scale 📎☞

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Students:

- solve problems involving simulations or trials of experiments in a variety of contexts **AAM** 📎💻
  - identify factors that could complicate the simulation of real-world events (ACMEM153)
  - use relative frequencies obtained from data as point estimates of probabilities (ACMMM055)

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Students:
- use arrays and tree diagrams to determine the outcomes and probabilities for multi-stage experiments (ACMEM156) **AAM** 📎

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Students:
- use Venn diagrams, set language and notation for events, including $\bar{A}$ (or A' or A^c) for the complement of an event A, $A \cap B$ for 'A and B', the intersection of events A and B, and $A \cup B$ for 'A or B', the union of events A and B, and recognise mutually exclusive events (ACMMM050) **AAM**
    - use everyday occurrences to illustrate set descriptions and representations of events and set operations (ACMMM051)

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Students:

- use the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054) **AAM** 📎

Students:

- understand the notion of conditional probability and recognise and use language that indicates conditionality (ACMMM056)

Students:

- use the notation $P(A|B)$ and the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$ for conditional probability (ACMMM057) **AAM**

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Students:

- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)

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Students:

- use the multiplication law $P(A \cap B) = P(A)P(B)$ for independent events A and B and recognise the symmetry of independence in simple probability situations (ACMMM059)

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Students:

S1.2: Discrete probability distributions

- define and categorise random variables
  - know that a random variable describes some aspect in a population from which samples can be drawn
  - know the difference between a discrete random variable and a continuous random variable

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Students:
- use discrete random variables and associated probabilities to solve practical problems (ACMMM142) **AAM**
  - use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable (ACMMM137)
  - recognise uniform random variables and use them to model random phenomena with equally likely outcomes (ACMMM138)
  - examine simple examples of non-uniform discrete random variables, and recognise that for any random variable, X, the sum of the probabilities is 1 (ACMMM139)
  - recognise the mean or expected value, $E(X) = \mu$, of a discrete random variable X as a measure of centre, and evaluate it in simple cases (ACMMM140)
  - recognise the variance, $\mathrm{Var}(X)$, and standard deviation (σ) of a discrete random variable as measures of spread, and evaluate them in simple cases (ACMMM141)
  - use $\mathrm{Var}(X) = E((X - \mu)^2) = E(X^2) - \mu^2$ for a continuous random variable and $\mathrm{Var}(x) = \sigma^2$ for a dataset

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Students:

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Students:
- understand that a sample mean, $\bar{x}$, is an estimate of the associated population mean μ, and that the sample standard deviation, s, is an estimate of the associated population standard deviation, σ, and that these estimates get better as the sample size increases and when we have independent observations

Students: